WORKSHOP ON COMPUTABILITY THEORY
AND
MIND, MECHANISM AND MATHEMATICS

FACULTY OF MATHEMATICS AND COMPUTER SCIENCE,
UNIVERSITY OF BUCHAREST,
14 ACADEMIEI ST., ROOM 0, GROUND FLOOR,
JUNE 27-28, 2015

SATURDAY, JUNE 27

9:20am WELCOME
9:30am STEPHANIE DICK (Harvard University)
Implementation and Epistemology in Early Computer Mathematics
10:30am ANDRE NIES (University of Auckland)
Objects of Greatest Complexity in a Class
11:00am BREAK
11:30am NIKOLAY BAZHENOV (Sobolev Institute of Mathematics)
Effective Categoricity for Decidable Structures
12:00pm PAUL GRANT (University of Cambridge)
Tuning of Signal Sending and Receiving for Bacterial Patterning Circuits
12:30pm LUNCH
2:00pm RUTGER KUYPER (Radboud University Nijmegen)
The Medvedev Lattice and First-Order Logic
2:30pm MARGARITA KOROVINA (Ershov Institute of Informatics Systems - presentation via Skype)
Index Sets in Computable Analysis
3:00pm BREAK
3:30pm STEFFEN LEMPP (University of Wisconsin-Madison)
Spectra of Computable Models of Strongly Minimal Disintegrated Theories
4:30pm SIMON MARTIEL (University of Nice-Sophia Antipolis)
Reversible Causal Graph Dynamics
5:00pm MARK BRAVERMAN (Princeton University - Presentation via Skype)
Information Complexity is Computable
5:30pm CLOSE
SUNDAY, JUNE 28

9:30am  STEVE SIMPSON (Pennsylvania State University)
        The Density Theorem for the Muchnik Lattice
10:30am ERIC ASTOR (University of Chicago)
        “Almost Everywhere” in the Natural Numbers: Intrinsic Density and
        Effective Negligibility
11:00am BREAK
11:30am NOAM GREENBERG (Victoria University of Wellington)
        Anti-Randomness and Lowness in Computability Theory
12:00pm JOE MILLER (University of Wisconsin-Madison)
        Subclasses of the K-Trivial Degrees
12:30pm LUNCH
2:00pm  DAVID GAMEZ (University of Sussex)
        How Can We Scientifically Study Consciousness?
2:30pm  ANDREY SARIEV (Sofia University)
        Definability in the Local Theory of the ω-Turing Degrees
3:00pm BREAK
3:30pm REBECCA SCHULMAN (Johns Hopkins University - Presentation via Skype)
        Ubiquitous Chemical Computation
4:30pm ALBERTO MARCONE (Università di Udine)
        Some Recent Advances in the Reverse Mathematics of Partial Orders
5:00pm ANDREW MARKS (Caltech)
        Amenability and Computability
5:30pm CLOSE
**ABSTRACTS**

**ERIC ASTOR, University of Chicago**

“Almost Everywhere” in the Natural Numbers: Intrinsic Density and Effective Negligibility

We investigate a computably-invariant restriction of asymptotic density, and observe that it has strong connections to both randomness and classical computability theory. In particular, we use it to define a new immunity property, recognize a new form of stochasticity, and find an unexpected connection to functions avoiding weak computable approximation. We then apply similar ideas to create computably-invariant restrictions of asymptotic computability, deriving from Jockusch and Schupp’s coarse and generic computabilities. Finally, we prove a generalization of Rice’s Theorem, giving a lower bound on the strength of these new notions of intrinsic asymptotic computability.

**NIKOLAY BAZHENOV, Sobolev Institute of Mathematics**

Effective Categoricity for Decidable Structures

In this talk we discuss the complexity of isomorphisms between a decidable structure and its computable copies. In particular, we review some recent results on SC-autostability spectra. Roughly speaking, the SC-autostability spectrum of a structure $A$ is the set of all Turing degrees capable of computing isomorphisms between arbitrary decidable copies of $A$. We will also talk about recent work with Sergey Goncharov and Margarita Marchuk on the complexity of index sets for SC-autostable structures.

**MARK BRAVERMAN, Princeton University**

Information Complexity is Computable

The information complexity of a function $f$ is the minimum amount of information Alice and Bob need to exchange to compute the function $f$. We provide an algorithm for approximating the information complexity of an arbitrary function $f$ to within any additive error $\epsilon > 0$. In the process, we give the first explicit upper bound on the rate of convergence of the information complexity of $f$ when restricted to $b$-bit protocols to the (unrestricted) information complexity of $f$.

**STEPHANIE DICK, Harvard University**

Implementation and Epistemology in Early Computer Mathematics

There is no automation without invention. Especially in the early decades, actually getting programs to run on computers was no small feat. Models and algorithms had to be translated into a form that computers could execute. Significant hurdles related to the allocation and management of memory had to be overcome. The affordances of computing machines had to be accommodated. Models and algorithms had to be implemented. The work of implementation spans multiple media - from paper to transistor - and involves many practices - from diagramming to coding. In implementing programs practitioners had to craft many new tools, both abstract and material. Implementation is where abstraction and materiality meet. It is the site where we see practitioners rethinking their objects of interest, their disciplines, their theories, their questions, through the lens of computing machines. And yet, where histories
of software even exist, their implementation is seldom a focus. Historians have, for the most part, emphasized high level description over low-level detail, taking for granted the complex division between hardware and software.

This paper focuses on implementation. In particular, I track how one early computing practitioner - Chinese American Hao Wang - took a highly abstract piece of mathematical logic and translated it into an actionable set of computer tools for mathematical research. I propose that in doing so, Wang introduced new tools and practices to mathematics. The resulting program, "The Program P" developed in the late 1950s, also reveals the epistemological significance of implementation: in making it, Wang came to know new things about logic and to know them in quite new ways.

DAVID GAMEZ, University of Sussex

How Can We Scientifically Study Consciousness?

People studying consciousness get repeatedly dragged back to basic questions about the nature of consciousness, the hard problem and whether computers are conscious. Some of these problems are linked to the tools that we use to study consciousness, such as imagination and thought experiments. Other problems are linked to the measurement of consciousness through first-person reports.

Scientific research on consciousness does not directly engage with these problems. But they are legitimate concerns that could invalidate scientific results. The endless inconclusive debates suggest that these problems cannot be solved. But we can make reasonable assumptions that explicitly set them aside. The results from the science of consciousness can then be considered to be true given these assumptions.

This talk will outline my approach to the scientific study of consciousness. This uses a framework of definitions and assumptions to neutralize most of the problems with consciousness. I will explain how we can accurately measure consciousness and the physical world, and develop mathematical theories of consciousness that can answer questions about the consciousness of brain-damaged patients, bats and robots.

PAUL GRANT, University of Cambridge

Tuning of Signal Sending and Receiving for Bacterial Patterning Circuits

The study of the development of multicellular organisms has yielded a multitude of mechanisms by which complex structures can arise from simple molecular and physical interactions. These self-organizing processes are a rich source of inspiration for the engineering of synthetic biological systems that arrange matter in ways that may prove useful for biomaterials, organized bioreactor communities, or medical applications. They can also serve as a testbed for the design rules needed to re-engineer multicellular systems such as crop plants to create novel morphologies and functionalities. In the short term, however, we can–by building synthetic systems based on the principles of development–test how well we understand those principles, explore the parameters under which they can function, and determine how generalizable they are to new contexts. To this end we have built a synthetic two-channel cell-cell communication system using two different quorum-sensing signals that can be sent and received by E. coli cells with minimal crosstalk. Because we have built this system from scratch we have full
control over the components allowing us to measure the response in a wide variety of conditions. This large amount of quantitative data allows us to infer the parameters to a highly detailed mathematical model giving us the ability to predict the optimal expression levels of receptors to minimize crosstalk. We have combined these signaling circuits with a novel spatial assay system in which cells are grown on gridded membranes so that we can create arbitrary arrangements of populations. In this context we have built a circuit that can propagate signals by positive feedback through mutual activation and a patterning circuit that creates mutually exclusive contiguous domains of gene expression with sharp boundaries by, analogously to the drosophila gap gene network, using self-activation combined with mutual inhibition to create bistability at the population level.

NOAM GREENBERG, Victoria University of Wellington

Anti-Randomness and Lowness in Computability Theory

This is an introduction to the talk by Joe Miller on subclasses of the $K$-trivial degrees. This is work in algorithmic randomness. In general we try to understand reals which are very close to being computable. I will discuss the background and, time permitting, some of the techniques involved.

MARGARITA KOROVINA, Ershov Institute of Informatics Systems

Index Sets in Computable Analysis

We report on the ongoing program of analysing the complexity of various problems in computable analysis in terms of the complexity of the associated index sets. In the framework of effectively enumerable topological spaces, we construct principal computable numberings of partial majorant-computable real-valued functions, co-effectively closed sets and calculate the complexity of index sets for important problems such as root verification and function equality. For example, we show that, for partial majorant-computable real functions, the equality problem is $\Pi^1_1$-complete. (joint work with Oleg Kudinov)

RUTGER KUYPER, Radboud University Nijmegen

The Medvedev Lattice and First-Order Logic

The Medvedev and Muchnik lattices are two lattices from computability theory that have been useful in various parts of computability theory. However, their original motivation comes from intuitionistic logic. Roughly speaking, intuitionistic logic is classical logic, except that the law of the excluded middle "p or not p" is not a valid principle. Kolmogorov interpreted intuitionistic logic in an informal way as a "calculus of problems", arguing that proving statements in intuitionistic logic is like calculating solutions to problems. The Medvedev and Muchnik lattices provide two possible formalisations of Kolmogorov's ideas.

However, the work of Medvedev and Muchnik only concerns propositional logic, while Kolmogorov also discussed how to deal with quantifiers. We extend the formalisations of Medvedev and Muchnik to first-order logic using techniques from categorical logic, yielding the hyperdoctrine of mass problems. Just like the Medvedev and Muchnik lattices, this hyperdoctrine is defined using familiar notions from computability theory, thus yielding us a computability-theoretic interpretation of intuitionistic first-order logic.
In this talk we will introduce the hyperdoctrine of mass problems. Furthermore, we will discuss some results about which intuitionistic first-order theories have models in this hyperdoctrine and which theories do not.

STEFFEN LEMPP, University of Wisconsin

Spectra of Computable Models of Strongly Minimal Disintegrated Theories

I will discuss on-going joint work with Uri Andrews in computable model theory.

The only general known upper bound on spectra of computable models of strongly minimal disintegrated theories is \( \Sigma^0_5 \) (by a result of Goncharov, Harizanov, Laskowski, McCoy and myself).

In a 2014 paper, Andrews and Medvedev completely characterized the possible spectra computable models of strongly minimal disintegrated theories in a finite language: There are only three, with \( \{0\} \) being the only nontrivial one.

Andrews and I are currently working on extending this to infinite computable (relational) languages.

For binary languages, we have completely identified all seven possible spectra (namely, in addition to the three above, also \( \{1\}, \{0,1\}, [1,\omega] \) and \( \{\omega\} \)).

For languages of bounded arity and theories in which all relations are at most Morley rank 1, we have completely identified all ten possible spectra (namely, in addition to the seven above, also \( \{0,\omega\}, \{1,\omega\} \) and \( \{0,1,\omega\} \)).

For languages of arbitrary arity and theories in which all relations are at most Morley rank 1, we have completely identified all possible spectra (namely, in addition to the ten above, also \( \{0,\alpha\} \) for any alpha less than or equal to \( \omega \)).

For ternary languages, we can show that there are at most eighteen and at least ten spectra.

We conjecture that in general, for any language of arity at most \( n \), there is only a finite number of possible spectra.

ALBERTO MARCONE, Università di Udine

Some Recent Advances in the Reverse Mathematics of Partial Orders

In this talk I will survey some results that deal with the reverse mathematics of theorems about (countable) partial orders and that were obtained in the last few years with Emanuele Frittaion. The theorems considered include results about the existence of linear extensions that preserve finiteness properties, the characterization of the partial orders such that any initial interval is a finite union of ideals, the characterization of the partial orders with countably many ideals. Some of the statements are equivalent to one of the ’big five’ systems (albeit some of proofs are nontrivial), while others exhibit interesting reverse mathematics behaviors: we have (the first?) example of an equivalence between a ”real mathematical theorem” and \( B - \Sigma^0_2 \), and a candidate for a principle of intermediate strength between WWKL\(_0\) and WKL\(_0\).

If time allows I will also mention some results about wqo and Noetherian spaces that will be dealt with in more detail in the talk by Paul Shafer at CiE.
ANDREW MARKS, Caltech

Amenability and Computability

TBA

SIMON MARTIEL, University of Nice-Sophia Antipolis

Reversible Causal Graph Dynamics

Causal Graph Dynamics extend Cellular Automata to arbitrary, bounded-degree, time-varying graphs. The whole graph evolves in discrete time steps, and this global evolution is required to have a number of physics-like symmetries: shift-invariance (it acts everywhere the same) and causality (information has a bounded speed of propagation). We add a further physics-like symmetry, namely reversibility.

JOE MILLER, University of Wisconsin

Subclasses of the $K$-Trivial Degrees

Noam Greenberg will give an introduction to the theory of $K$-triviality. I will follow up by talking about recent work with Greenberg and Nies on the fine structure of the class of K-trivial sets. We have isolated a natural dense family of subideals, including the sets that are computable from two relatively random sequences. I will talk about these ideals and the various characterizations we have found (including even more recent work with Turetsky).

ANDRE NIES, University of Auckland

Objects of greatest complexity in a class

The following type of question arises frequently in mathematics and theoretical computer science. We are given a class of objects and a method to compare their complexity. Is there a most complex object in the class? What are the properties of such objects? Is such an object in some sense unique?

For instance, the halting problem is a most complex computably enumerable set for many-one reducibility. SAT is a most complex NP set under polynomial time $m$-reducibility. Chaitin’s Omega is a most complex left-computably enumerable real under Solovay reducibility. Every countable graph is embedded into the Rado graph.

The complete objects in the first two examples have a lot of structure, while the ones in the last two examples are random. The halting problem and Omega are unique. For SAT uniqueness is an open question of Berman and Hartmanis.

I will discuss such examples in detail, and also attempt to develop a metatheory.


ANDREY SARIEV, Sofia University

Definability in the Local Theory of the $\omega$-Turing Degrees

The $\omega$-Turing degrees ($\mathcal{D}_\omega$) are an extension of the Turing degrees, which is induced by the following relation on sequences of sets of natural numbers:

$$\{A_k\}_{k<\omega} \leq_\omega \{B_k\}_{k<\omega} \text{ iff } J(\{B_k\}_{k<\omega}) \subseteq J(\{A_k\}_{k<\omega}),$$
where $J(\{C_k\}_{k<\omega}) = \{\text{deg}_T(X) \mid C_k \leq_T X^{(k)} \text{ uniformly in } k\}$ denotes the so called jump-class of the sequence $\{C_k\}_{k<\omega}$.

We introduce a jump $(\{A_k\}_{k<\omega})'$ of the sequence $\{A_k\}_{k<\omega}$ in such a way that $J(\{A_k\}_{k<\omega}') = \{x' \mid x \in J(\{A_k\}_{k<\omega})\}$. The jump on sequences is strictly monotone and preserves the order, so it induces a jump operation in the degree structure.

This jump operation has a least jump inversion property which allows to show that $\mathcal{D}_T$ is first-order definable in $\omega$-Turing degrees (with jump) and that the groups $\text{Aut}(\mathcal{D}_T)$ and $\text{Aut}(\mathcal{D}_T')$ are isomorphic.

The jump operation gives rise to the local substructure of the $\omega$-Turing degrees. We show that for each $n < \omega$, the classes $L_n$, $H_n$ and $I$ respectively of low$_n$, high$_n$ and intermediate degrees are first-order definable in the local substructure.

REBECCA SCHULMAN, Johns Hopkins University

Ubiquitous Chemical Computation

In living things, the interactions of molecules orchestrate complex self-organization, interpret stimuli from the environment, and produce adaptive responses. Yet how molecules, which alone have little capacity for information processing or memory can collectively orchestrate these functions has been a longstanding mystery.

In one of his last major publications, Turing described how simple sets of molecules could form well-defined spatial patterns within a simple homogeneous medium, a process he believed could explain the initial steps of biological development and patterning. Recently both an increasingly mechanistic understanding of many cellular processes and an ability to engineer molecules that can process information has made it possible to re-examine how chemicals can compute and to also then build new types of materials with the intrinsic capacity for computation and planning. I will describe recent advances in this area along with work in my group for designing a new set of materials capable of self-patterning, moving, and interacting with their environment that function by using trillions of simple molecular computers that are dispersed throughout a system.

STEVE SIMPSON, Pennsylvania State University

The Density Theorem for the Muchnik Lattice

Let $\mathcal{E}_T$ be the upper semilattice of recursively enumerable Turing degrees. There is a strong analogy between $\mathcal{E}_T$ and $\mathcal{E}_w$, the lattice of Muchnik degrees of nonempty $\Pi^0_1$ subsets of $\{0,1\}^\mathbb{N}$. The most famous structural theorems concerning $\mathcal{E}_T$ are the Splitting Theorem (Sacks 1962) and the Density Theorem (Sacks 1964). The analogous Splitting Theorem for $\mathcal{E}_w$ was proved by Stephen Binns and published in 2003. The analogous Density Theorem for $\mathcal{E}_w$ has been a long-standing open question, but it was recently proved by Binns, Shore, and Simpson and will appear in a forthcoming paper. In this talk I shall sketch the proof of the Density Theorem for $\mathcal{E}_w$. The proof involves a small amount of hyperarithmetical theory.