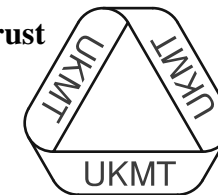


The United Kingdom Mathematics Trust



**Intermediate Mathematical Olympiad and Kangaroo  
(IMOK)**

**Olympiad Maclaurin Paper**

Thursday 19th March 2009

All candidates must be in *School Year 11* (England and Wales), *S4* (Scotland), or *School Year 12* (Northern Ireland).

**READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING**

1. Time allowed: 2 hours.
2. **The use of calculators, protractors and squared paper is forbidden.**  
Rulers and compasses may be used.
3. Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Cover Sheet on top.
4. Start each question on a fresh A4 sheet.  
You may wish to work in rough first, then set out your final solution with clear explanations and proofs.  
***Do not hand in rough work.***
5. Answers must be FULLY SIMPLIFIED, and EXACT. They may contain symbols such as  $\pi$ , fractions, or square roots, if appropriate, but NOT decimal approximations.
6. Give full written solutions, including mathematical reasons as to why your method is correct. Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.
7. **These problems are meant to be challenging!** The earlier questions tend to be easier; the last two questions are the most demanding.  
Do not hurry, but spend time working carefully on one question before attempting another.  
Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

**DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!**

The United Kingdom Mathematics Trust is a Registered Charity.

*Enquiries should be sent to: Maths Challenges Office,*

*School of Mathematics, University of Leeds, Leeds, LS2 9JT.*

*(Tel. 0113 343 2339)*

*<http://www.ukmt.org.uk>*

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- Just stating an answer, even a correct one, will earn you very few marks.
- Incomplete or poorly presented solutions will not receive full marks.
- Do not hand in rough work.

1. Five numbers are arranged in increasing order. As they get larger the difference between adjacent numbers doubles.

The average of the five numbers is 11 more than the middle number. The sum of the second and fourth numbers is equal to the largest number.

What is the largest number?

2. Miko always remembers his four-digit PIN (personal identification number) because
  - (a) it is a perfect square, and
  - (b) it has the property that, when it is divided by 2, or 3, or 4, or 5, or 6, or 7, or 8, or 9, there is always a remainder of 1.

What is Miko's PIN?

3. Solve the simultaneous equations

$$\frac{5xy}{x+y} = 6$$

$$\frac{4xz}{x+z} = 3$$

$$\frac{3yz}{y+z} = 2.$$

4. In a trapezium  $ABCD$  the sides  $AB$  and  $DC$  are parallel and  $\angle BAD = \angle ABC < 90^\circ$ . Point  $P$  lies on  $AB$  with  $\angle CPD = \angle BAD$ .

Prove that  $PC^2 + PD^2 = AB \times DC$ .

5. A lottery involves five balls being selected from a drum. Each ball has a different positive integer printed on it. Show that, whichever five balls are selected, it is always possible to choose three of them so that the sum of the numbers on these three balls is a multiple of 3.

6. In the figure,  $p$ ,  $q$ ,  $r$  and  $s$  are the lengths of four arcs which together form the circumference of the circle. Find, in simplified form, an expression for  $s$  in terms of  $p$ ,  $q$  and  $r$ .

