The United Kingdom Mathematics Trust

Intermediate Mathematical Olympiad and Kangaroo (IMOK)

Olympiad Hamilton Paper

Thursday 19th March 2009

All candidates must be in *School Year 10* (England and Wales), *S3* (Scotland), or *School Year 11* (Northern Ireland).

READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators, protractors and squared paper is forbidden.**
   Rulers and compasses may be used.
3. Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Cover Sheet on top.
4. Start each question on a fresh A4 sheet.
   You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

   Do not hand in rough work.
5. Answers must be FULLY SIMPLIFIED, and EXACT. They may contain symbols such as π, fractions, or square roots, if appropriate, but NOT decimal approximations.
6. Give full written solutions, including mathematical reasons as to why your method is correct.
   Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.
7. **These problems are meant to be challenging!** The earlier questions tend to be easier; the last two questions are the most demanding.
   Do not hurry, but spend time working carefully on one question before attempting another.
   Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

The United Kingdom Mathematics Trust is a Registered Charity.
Enquiries should be sent to: Maths Challenges Office,
School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

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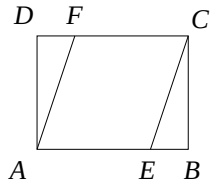
1. An aquarium contains 280 tropical fish of various kinds. If 60 more clownfish were added to the aquarium, the proportion of clownfish would be doubled.

How many clownfish are in the aquarium?

2. Find the possible values of the digits p and q, given that the five-digit number '$p543q$' is a multiple of 36.

3. In the diagram, $ABCD$ is a rectangle with $AB = 16$ cm and $BC = 12$ cm. Points E and F lie on sides AB and CD so that $AECF$ is a rhombus.

What is the length of EF?



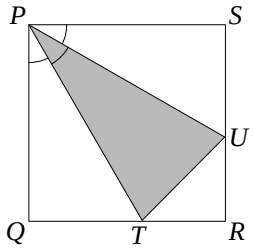
4. Four positive integers a, b, c and d are such that:

the sum of a and b is half the sum of c and d;
the sum of a and c is twice the sum of b and d;
the sum of a and d is one and a half times the sum of b and c.

What is the smallest possible value of $a + b + c + d$?

5. The diagram shows a triangle PTU inscribed in a square $PQRS$. Each of the marked angles at P is equal to 30°.

Prove that the area of the triangle PTU is one third of the area of the square $PQRS$.



6. Two different cuboids are placed together, face-to-face, to form a large cuboid.

The surface area of the large cuboid is $\frac{3}{4}$ of the total surface area of the original two cuboids.

Prove that the lengths of the edges of the large cuboid may be labelled x, y and z, where

$$\frac{2}{z} = \frac{1}{x} + \frac{1}{y}.$$