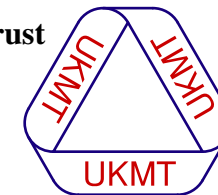


The United Kingdom Mathematics Trust



**Intermediate Mathematical Olympiad and Kangaroo  
(IMOK)**

**Olympiad Maclaurin Paper**

Thursday 17th March 2011

All candidates must be in *School Year 11* (England and Wales), *S4* (Scotland), or  
*School Year 12* (Northern Ireland).

**READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING**

1. Time allowed: 2 hours.
2. **The use of calculators, protractors and squared paper is forbidden.**  
Rulers and compasses may be used.
3. Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Cover Sheet on top.
4. Start each question on a fresh A4 sheet.  
You may wish to work in rough first, then set out your final solution with clear explanations and proofs.  
***Do not hand in rough work.***
5. Answers must be FULLY SIMPLIFIED, and EXACT. They may contain symbols such as  $\pi$ , fractions, or square roots, if appropriate, but NOT decimal approximations.
6. Give full written solutions, including mathematical reasons as to why your method is correct. Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.
7. **These problems are meant to be challenging!** The earlier questions tend to be easier; the last two questions are the most demanding.  
Do not hurry, but spend time working carefully on one question before attempting another.  
Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

**DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!**

The United Kingdom Mathematics Trust is a Registered Charity.

*Enquiries should be sent to: Maths Challenges Office,*

*School of Maths Satellite, University of Leeds, Leeds, LS2 9JT.*

*(Tel. 0113 343 2339)*

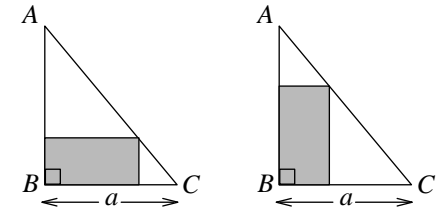
*<http://www.ukmt.org.uk>*

- Do not hurry, but spend time working carefully on one question before attempting another.
- Try to finish whole questions even if you cannot do many.
- You will have done well if you hand in full solutions to two or more questions.
- Answers must be FULLY SIMPLIFIED, and EXACT. They may contain symbols such as  $\pi$ , fractions, or square roots, if appropriate, but NOT decimal approximations.
- Give full written solutions, including mathematical reasons as to why your method is correct.
- Just stating an answer, even a correct one, will earn you very few marks.
- Incomplete or poorly presented solutions will not receive full marks.
- Do not hand in rough work.

1. How many positive integers leave a remainder of 31 when divided into 2011?

2. I have 44 socks in my drawer, each either red or black. In the dark I randomly pick two socks, and the probability that they do not match is  $\frac{192}{473}$ . How many of the 44 socks are red?

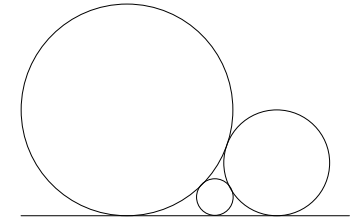
3. The diagrams show a rectangle that just fits inside right-angled triangle  $ABC$  in two different ways. One side of the triangle has length  $a$ . Prove that the perimeter of the rectangle is  $2a$ .



4. How many solutions are there to the equation  $x^2 + y^2 = x^3$ , where  $x$  and  $y$  are positive integers and  $x$  is less than 2011?

5. Three circles touch the same straight line and touch each other, as shown. Prove that the radii  $a$ ,  $b$  and  $c$ , where  $c$  is smallest, satisfy the equation

$$\frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}} = \frac{1}{\sqrt{c}}$$



6. The numbers 1 to 9 are placed in the cells of a  $3 \times 3$  square grid, one to each cell. In each of the four  $2 \times 2$  blocks of adjacent cells, such as the one shaded, the four numbers have the same total  $T$ . What is the maximum possible value of  $T$ ?

