

UK Junior Mathematical Olympiad 2008

Organised by The United Kingdom Mathematics Trust

Tuesday 17th June 2008

RULES AND GUIDELINES : READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators, measuring instruments and squared paper is forbidden.**
3. All candidates must be in *School Year 8 or below* (England and Wales), *S2 or below* (Scotland), *School Year 9 or below* (Northern Ireland).
4. For questions in Section A *only the answer is required*. Enter each answer neatly in the relevant box on the Front Sheet. Do not hand in rough work. Write in blue or black pen or pencil.

For questions in Section B you must give *full written solutions*, including clear mathematical explanations as to why your method is correct.

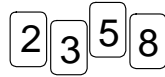
Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Front Sheet on top.

Do not hand in rough work.
5. Questions A1-A10 are relatively short questions. Try to complete Section A within the first 45 minutes so as to allow well over an hour for Section B.
6. Questions B1-B6 are longer questions requiring *full written solutions*. This means that each answer must be accompanied by clear explanations and proofs. Work in rough first, then set out your final solution with clear explanations of each step.
7. These problems are meant to be challenging! Do not hurry. Try the earlier questions in each section first (they tend to be easier). Try to finish whole questions even if you can't do many. A good candidate will have done most of Section A and given solutions to at least two questions in Section B.
8. Answers must be FULLY SIMPLIFIED, and EXACT using symbols like π , fractions, or square roots if appropriate, but NOT decimal approximations.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

Section A

- A1** In how many ways is it possible to place side by side two of the cards shown to form a two-digit prime number?

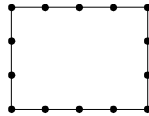


- A2** Tony wants to form a square with perimeter 12 cm by folding a rectangle in half and then in half again. What is the maximum possible perimeter of the original rectangle?

- A3** Given that $\frac{1}{2} + \frac{1}{3} + \frac{1}{12} + \frac{1}{18} + \frac{1}{x} = 1$, what is the value of x ?

- A4** How many three-digit numbers have the product of their digits equal to 6?

- A5** A 3 by 4 rectangle has 14 points equally spaced around its four sides, as shown.



In how many ways is it possible to join two of the points by a straight line so that the rectangle is divided into two parts which have areas in the ratio 1 : 3?

- A6** How many positive square numbers are factors of 1600?

- A7** In a *Magic Square*, the sum of the three numbers in each row, each column and each of the two main diagonals is the same.

What is the value of x in the partially completed magic square shown?

		6
x	4	5

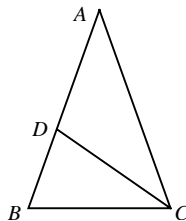
- A8** Granny shares a packet of sweets between her four granddaughters. The girls, Clarrie, Lizzie, Annie and Danni, always in that order, each take 8 sweets in turn, over and over again until, finally, there are some sweets left for Danni, but there are fewer than 8. Danni takes all the sweets that are left. The other three girls then give Danni some of their sweets so that all four girls have the same number of sweets.

How many sweets does each of the other three granddaughters give to Danni?

- A9** In the diagram, CD is the bisector of angle ACB .

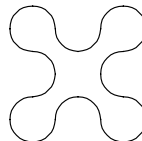
Also, $BC = CD$ and $AB = AC$.

What is the size of angle CDA ?



- A10** The perimeter of the shape shown on the right is made from 20 quarter-circles, each with radius 2 cm.

What is the area of the shape?



Section B

Your solutions to Section B will have a major effect on your JMO results. Concentrate on one or two questions first and then **write out full solutions** (not just brief 'answers').

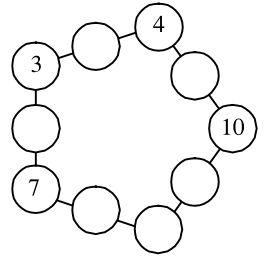
- B1** Tamsin has a selection of cubical boxes whose internal dimensions are whole numbers of centimetres, that is, $1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm}$, $2 \text{ cm} \times 2 \text{ cm} \times 2 \text{ cm}$, and so on.

What are the dimensions of the smallest of these boxes in which Tamsin could fit ten rectangular blocks each measuring $3 \text{ cm} \times 2 \text{ cm} \times 1 \text{ cm}$ without the blocks extending outside the box?

- B2** Each of the numbers from 1 to 10 is to be placed in the circles so that the sum of each line of three numbers is equal to T .

Four numbers have already been entered.

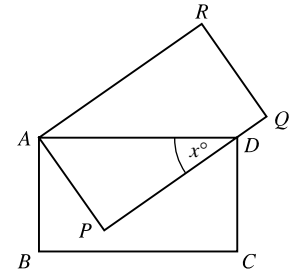
Find all the possible values of T .



- B3** In the diagram $ABCD$ and $APQR$ are congruent rectangles.

The side PQ passes through the point D and $\angle PDA = x^\circ$.

Find an expression for $\angle DRQ$ in terms of x .

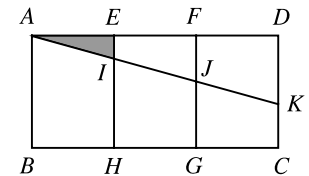


- B4** For each positive two-digit number, Jack subtracts the units digit from the tens digit; for example, the number 34 gives $3 - 4 = -1$.

What is the sum of all his results?

- B5** In the diagram, the rectangle $ABCD$ is divided into three congruent rectangles. The line segment JK divides $CDFG$ into two parts of equal area.

What is the area of triangle AEI as a fraction of the area of $ABCD$?



- B6** In a sequence of positive integers, each term is larger than the previous term. Also, after the first two terms, each term is the sum of the previous two terms.

The eighth term of the sequence is 390. What is the ninth term?