UKMT MENTORING SCHEME (Intermediate Level)
April 2011 (Sheet 7)
Questions

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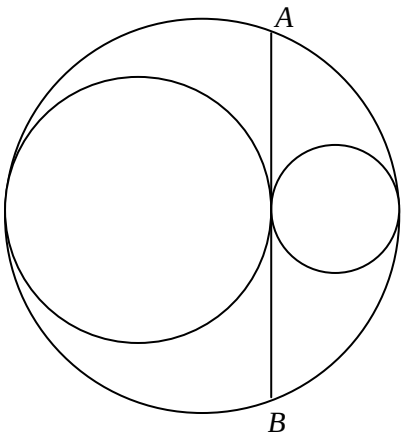
1. If $a - 3b - 9c + 27d = 19$, and all of a, b, c and d have values of 1, 2 or 3, find the values of a, b, c and d.

2. When each digit of a four-digit perfect square is increased by three, the result is another four-digit perfect square. Find all four-digit perfect squares which have this property.



3. Three circles are mutually touching at three collinear points as shown. The chord AB is a tangent to the two smaller circles. If $AB = 8$, find the area which lies within the large circle but outside the two smaller circles.

4. Find all positive integers n such that $n^4 + n^2 - 2$ is divisible by 72.

5. Two intersecting circles C_1 and C_2 have a common tangent which touches C_1 at P and C_2 at Q. The two circles intersect at M and N, where N is closer to PQ than M is. Prove that the triangles MNP and MNQ have equal areas.

6. Let n be the number $2^{31} \times 3^{19}$. How many positive integer factors of n^2 are less than n but are not factors of n?

7. Each cell in a 5×41 grid is coloured either red or blue. Prove that some 3 rows and 3 columns must intersect in 9 cells of the same colour.

8. Determine all triples (a, b, c) of integers such that $1 \le a \le b \le c$ and

$$\frac{1}{a+2} + \frac{1}{b+2} = \frac{1}{2} + \frac{1}{c+2}.$$

Deadline for receipt of solutions: 30th April 2011

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