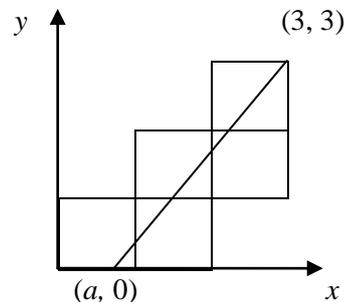


*I hope you enjoyed doing the October sheet. Again the questions are graded so that the first few are quite accessible with a little bit of thought, but the later ones are more taxing.*

1. Five unit squares are drawn on a coordinate grid as shown. A line from the point  $(3, 3)$  to the point  $(a, 0)$  exactly divides the area of the five squares into two equal areas. Work out the value of  $a$ .



2. a) How many positive integers up to 2010 are divisible by either 3 or 5?  
 b) How many positive integers up to 2010 are divisible by either 3, 5 or 7?
3. The numbers  $9 = 2 + 3 + 4$  and  $10 = 1 + 2 + 3 + 4$  are each expressible as the sum of (more than one!) consecutive positive integers. Which of 2009, 2010, 404, 6464, 8192 are expressible in the same way? State and prove a general result as to precisely which numbers can be expressed like this.
4.  $ABCD$  is a convex quadrilateral such that  $AB = 9$  and  $CD = 12$ . Diagonals  $AC$  and  $BD$  intersect at  $E$ .  $AC = 14$  and triangles  $AED$  and  $BEC$  have equal areas. Calculate the length  $AE$ .  
 [Note: A convex quadrilateral is one where none of the internal angles is more than  $180^\circ$ . Or, to put it another way, if you put an elastic band round the outside, it would go along all the sides.]
5. A circle touches the positive  $x$ - and  $y$ -axes and is externally tangential to the circle centred at  $(3, 0)$  with radius 1. Find the radii of all possible such circles.
6. Some cells of an infinite square grid are coloured black, and the rest are white so that each  $2 \times 3$  or  $3 \times 2$  rectangle contains exactly 2 black cells. How many black cells could a  $9 \times 11$  rectangle contain?
7. a) How many values is it possible to make using three different British coins?  
 b) How many different subsets of the set  $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$  contain three elements and no pairs of consecutive integers?  
 c) Explain why the answers to (a) and (b) are the same (i.e. why they can be done by the same calculation, and it is not just coincidence that the answers are the same).
8. Find all positive integers  $m, n$  where  $n$  is odd such that  $\frac{1}{m} + \frac{4}{n} = \frac{1}{12}$ .

**Deadline for receipt of solutions: 29<sup>th</sup> November 2010**

Supported by Man Group plc Charitable Trust